

## Integration

### EXERCISE 5.1 [PAGE 119]

#### Exercise 5.1 | Q 1 | Page 119

Evaluate  $\int \frac{-2}{\sqrt{5x-4} - \sqrt{5x-2}} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{-2}{\sqrt{5x-4} - \sqrt{5x-2}} dx \\&= -2 \int \frac{1}{\sqrt{5x-4} - \sqrt{5x-2}} \times \frac{\sqrt{5x-4} + \sqrt{5x-2}}{\sqrt{5x-4} + \sqrt{5x-2}} dx \\&= -2 \int \frac{\sqrt{5x-4} + \sqrt{5x-2}}{(5x-4) - (5x-2)} dx \\&= -2 \int \frac{\sqrt{5x-4} + \sqrt{5x-2}}{-2} dx \\&= \int \left[ (5x-4)^{\frac{1}{2}} + (5x-2)^{\frac{1}{2}} \right] dx \\&= \int (5x-4)^{\frac{1}{2}} dx + \int (5x-2)^{\frac{1}{2}} dx \\&= \frac{(5x-4)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{5} + \frac{(5x-2)^{\frac{3}{2}}}{\frac{3}{2}} \times \frac{1}{5} + c \\&\therefore I = \frac{2}{15} \left[ (5x-4)^{\frac{3}{2}} + (5x-2)^{\frac{3}{2}} \right] + c\end{aligned}$$

#### Exercise 5.1 | Q 2 | Page 119

Evaluate  $\int \left( 1 + x + \frac{x^2}{2!} \right) dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \left( 1 + x + \frac{x^2}{2!} \right) dx \\ &= \int 1 \cdot dx + \int x \cdot dx + \frac{1}{2} \int x^2 \cdot dx \quad \dots [\because 2! = 2] \\ &= x + x^2 + \frac{1}{2} \cdot \frac{x^3}{3} + c \\ \therefore I &= x + \frac{x^2}{2} + \frac{x^3}{6} + c\end{aligned}$$

**Exercise 5.1 | Q 3 | Page 119**

Evaluate  $\int \frac{3x^3 - 2\sqrt{x}}{x} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{3x^3 - 2\sqrt{x}}{x} dx \\ &= \int \left( \frac{3x^3}{x} - \frac{2x^{\frac{1}{2}}}{x} \right) dx \\ &= \int \left( 3x^2 - 2x^{-\frac{1}{2}} \right) dx \\ &= 3 \int x^2 \cdot dx - 2 \int x^{-\frac{1}{2}} \cdot dx \\ &= 3 \left( \frac{x^3}{3} \right) - 2 \left( \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right) + c \\ \therefore I &= x^3 - 4\sqrt{x} + c\end{aligned}$$

**Exercise 5.1 | Q 4 | Page 119**

Evaluate  $\int (3x^2 - 5)^2 dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int (3x^2 - 5)^2 dx \\&= \int (9x^4 - 30x^2 + 25) dx \\&= 9 \int x^4 dx - 30 \int x^2 dx + 25 \int dx \\&= 9 \left( \frac{x^5}{5} \right) - 30 \left( \frac{x^3}{3} \right) + 25x + c \\ \therefore I &= \frac{9}{5}x^5 - 10x^3 + 25x + c\end{aligned}$$

**Exercise 5.1 | Q 5 | Page 119**

Evaluate  $\int \frac{1}{x(x-1)} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{1}{x(x-1)} dx \\&= \int \frac{x - x + 1}{x(x-1)} dx \\&= \int \frac{x - (x-1)}{x(x-1)} dx \\&= \int \left( \frac{1}{x-1} - \frac{1}{x} \right) dx\end{aligned}$$

$$\begin{aligned}
 &= \int \frac{1}{x-1} dx - \int \frac{1}{x} dx \\
 &= \log|x-1| - \log|x| + c \\
 \therefore I &= \log \left| \frac{x-1}{x} \right| + c
 \end{aligned}$$

### Exercise 5.1 | Q 6 | Page 119

If  $f'(x) = x^2 + 5$  and  $f(0) = -1$ , then find the value of  $f(x)$ .

**Solution:**

$$f'(x) = x^2 + 5 \quad \dots[\text{Given}]$$

$$f(x) = \int f'(x) dx$$

$$= \int (x^2 + 5) dx$$

$$= \int x^2 dx + 5 \int dx$$

$$\therefore f(x) = \frac{x^3}{3} + 5x + c \quad \dots(i)$$

$$\text{Now } f(0) = -1 \quad \dots[\text{Given}]$$

$$\therefore \frac{0^3}{3} + 5(0) + c = -1$$

$$\therefore c = -1$$

Substituting  $c = -1$  in (i), we get

$$f(x) = \frac{x^3}{3} + 5x - 1$$

### Exercise 5.1 | Q 7 | Page 119

If  $f'(x) = 4x^3 - 3x^2 + 2x + k$ ,  $f(0) = 1$  and  $f(1) = 4$ , find  $f(x)$ .

**Solution:**  $f'(x) = 4x^3 - 3x^2 + 2x + k \quad \dots[\text{Given}]$

$$f(x) = \int f'(x) dx$$

$$\begin{aligned}
 &= \int (4x^3 - 3x^2 + 2x + k) \, dx \\
 &= 4 \int x^3 \, dx - 3 \int x^2 \, dx + 2 \int x \, dx + k \int 1 \, dx \\
 &= 4 \left( \frac{x^4}{4} \right) - 3 \left( \frac{x^3}{3} \right) + 2 \left( \frac{x^2}{2} \right) + kx + c
 \end{aligned}$$

$$\therefore f(x) = x^4 - x^3 + x^2 + kx + c \quad \dots(i)$$

$$\text{Now, } f(0) = 1 \quad \dots[\text{Given}]$$

$$\therefore (0)^4 - (0)^3 + (0)^2 + k(0) + c = 1$$

$$\therefore c = 1 \quad \dots(ii)$$

$$\text{Also, } f(1) = 4$$

$$\therefore 1^4 - 1^3 + 1^2 + k(1) + 1 = 4$$

$$\therefore 2 + k = 4$$

$$\therefore k = 2 \quad \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$f(x) = x^4 - x^3 + x^2 + 2x + 1$$

#### Exercise 5.1 | Q 8 | Page 119

If  $f'(x) = \frac{x^2}{2} - kx + 1$ ,  $f(0) = 2$  and  $f(3) = 5$ , find  $f(x)$ .

**Solution:**

$$f'(x) = \frac{x^2}{2} - kx + 1 \quad \dots[\text{Given}]$$

$$f(x) = \int f'(x) \, dx$$

$$\begin{aligned}
&= \int \left( \frac{x^2}{2} - kx + 1 \right) dx \\
&= \frac{1}{2} \int x^2 dx - k \int x dx + \int 1 \cdot dx \\
&= \frac{1}{2} \cdot \frac{x^3}{3} - k \left( \frac{x^2}{2} \right) + x + c \\
\therefore f(x) &= \frac{x^3}{6} - \frac{k}{2} x^2 + x + c \quad \dots(i)
\end{aligned}$$

Now,  $f(0) = 2$

$$\therefore \frac{(0)^3}{6} - \frac{k}{2}(0)^2 + 0 + c = 2$$

$$\therefore c = 2 \quad \dots(ii)$$

Also  $f(3) = 5 \quad \dots[\text{Given}]$

$$\therefore \frac{(3)^3}{6} - \frac{k}{2}(3)^2 + 3 + 2 = 5$$

$$\therefore \frac{27}{6} - \frac{9k}{2} + 5 = 5$$

$$\therefore \frac{9}{2} - \frac{9k}{2} = 0$$

$$\therefore \frac{9k}{2} = \frac{9}{2}$$

$$\therefore k = 1 \quad \dots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$f(x) = \frac{x^3}{6} - \frac{x^2}{2} + x + 2$$

## EXERCISE 5.2 [PAGES 122 - 123]

### Exercise 5.2 | Q 1 | Page 122

**Evaluate the following.**

$$\int x\sqrt{1+x^2} \, dx$$

**Solution:**

$$\text{Let } I = \int x\sqrt{1+x^2} \, dx$$

$$\text{Put } 1+x^2 = t$$

$$\therefore 2x \cdot dx = dt$$

$$\therefore x \cdot dx = \frac{1}{2} dt$$

$$\therefore I = \frac{1}{2} \int \sqrt{t} \cdot dt$$

$$= \frac{1}{2} \int t^{\frac{1}{2}} \cdot dt$$

$$= \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{3} t^{\frac{3}{2}} + C$$

$$\therefore I = \frac{1}{3} (1+x^2)^{\frac{3}{2}} + C$$

### Exercise 5.2 | Q 2 | Page 123

**Evaluate the following.**

$$\int \frac{x^3}{\sqrt{1+x^4}} \, dx$$

**Solution:**

$$\text{Let } I = \int \frac{x^3}{\sqrt{1+x^4}} dx$$

$$\text{Put } 1 + x^4 = t$$

$$\therefore 4x^3 \cdot dx = dt$$

$$\therefore x^3 \cdot dx = \frac{1}{4} dt$$

$$\therefore I = \frac{1}{4} \int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{4} \int t^{-\frac{1}{2}} dt$$

$$= \frac{1}{4} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= \frac{1}{2} \sqrt{t} + c$$

$$\therefore I = \frac{1}{2} \sqrt{1+x^4} + c$$

**Exercise 5.2 | Q 3 | Page 123**

**Evaluate the following.**

$$\int (e^x + e^{-x})^2 (e^x - e^{-x}) dx$$

**Solution:**

$$\text{Let } I = \int (e^x + e^{-x})^2 \cdot (e^x - e^{-x}) dx$$

$$\text{Put } e^x + e^{-x} = t$$

$$\therefore (e^x + e^{-x}) dx = dt$$



$$\begin{aligned}
 \therefore I &= \int t^2 \cdot dt \\
 &= \frac{t^3}{3} + c \\
 \therefore I &= \frac{1}{3} (e^x + e^{-x})^3 + c
 \end{aligned}$$

**Exercise 5.2 | Q 4 | Page 123**

**Evaluate the following.**

$$\int \frac{1+x}{x+e^{-x}} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1+x}{x+e^{-x}} dx$$

$$= \int \frac{1+x}{x + \frac{1}{e^x}} dx$$

$$= \int \frac{1+x}{\frac{x \cdot e^x + 1}{e^x}} dx$$

$$= \int \frac{e^x(1+x)}{x \cdot e^x + 1} dx$$

$$\text{Put } x \cdot e^x + 1 = t$$

$$\therefore [x \cdot (e^x) + e^x(1) + 0]dx = dt$$

$$\therefore e^x(x+1)dx = dt$$

$$\therefore I = \int \frac{dt}{t}$$

$$= \log |t| + c$$

$$\therefore I = \log |x \cdot e^x + 1| + c$$

**Exercise 5.2 | Q 5 | Page 123**

**Evaluate the following.**

$$\int (x + 1)(x + 2)^7 (x + 3)dx$$

**Solution:** Let  $I = \int (x + 1)(x + 2)^7 (x + 3)dx$

Put  $x + 2 = t$

$$\therefore dx = dt$$

Also,  $x = t - 2$

$$\therefore x + 1 = t - 2 + 1$$

$$= t - 1$$

and  $x + 3 = t - 2 + 3$

$$= t + 1$$

$$\therefore I = \int (t - 1) \cdot t^7 (t + 1) \cdot dt$$

$$= \int (t^2 - 1) \cdot t^7 \cdot dt$$

$$= \int (t^9 - t^7) dt$$

$$= \int t^9 dt - \int t^7 dt$$

$$= \frac{t^{10}}{10} - \frac{t^8}{8} + c$$

$$\therefore I = \frac{(x + 2)^{10}}{10} - \frac{(x + 2)^8}{8} + c$$

**Exercise 5.2 | Q 6 | Page 123**

**Evaluate the following.**

$$\int \frac{1}{x \log x} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1}{x \log x} dx$$

$$\text{Put } \log x = t$$

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int \frac{1}{t} dt = \log |t| + c$$

$$\therefore I = \log |\log x| + c$$

**Alternate Method:**

$$\text{Let } I = \int \frac{1}{x \cdot \log x} dx$$

$$= \int \frac{1/x dx}{\log x}$$

$$\therefore I = \log |\log x| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

**Exercise 5.2 | Q 7 | Page 123**

**Evaluate the following.**

$$\int \frac{x^5}{x^2 + 1} dx$$

**Solution:**

$$\text{Let } I = \int \frac{x^5}{x^2 + 1} dx$$

$$\int \frac{(x^2)^2 \cdot x}{x^2 + 1} dx$$

$$\text{Put } x^2 + 1 = t$$

$$\therefore 2x \cdot dx = dt$$

$$\therefore x \cdot dx = \frac{1}{2} \cdot dt$$

$$\text{Also, } x^2 = t - 1$$

$$\therefore I = \int \frac{(t - 1)^2}{t} \cdot \frac{1}{2} dt$$

$$= \frac{1}{2} \int \frac{t^2 - 2t + 1}{t} dt$$

$$= \frac{1}{2} \int \left( t - 2 + \frac{1}{t} \right) dt$$

$$= \frac{1}{2} \left[ \frac{t^2}{2} - 2t + \log|t| \right] + c$$

$$= \frac{1}{4} t^2 - t + \frac{1}{2} \log|t| + c$$

$$\therefore I = \frac{1}{4} (x^2 + 1)^2 - (x^2 + 1) + \frac{1}{2} \log|x^2 + 1| + c$$

**Exercise 5.2 | Q 8 | Page 123**

**Evaluate the following.**

$$\int \frac{2x + 6}{\sqrt{x^2 + 6x + 3}} dx$$

**Solution:**

$$\text{Let } I = \int \frac{2x + 6}{\sqrt{x^2 + 6x + 3}} dx$$

$$\text{Put } x^2 + 6x + 3 = t$$

$$\therefore (2x + 6) dx = dt$$

$$\therefore I = \int \frac{dt}{\sqrt{t}}$$

$$= \int t^{-\frac{1}{2}} dt$$

$$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$= 2\sqrt{t} + c$$

$$\therefore I = 2\sqrt{x^2 + 6x + 3} + c$$

**Alternate Method:**

$$\text{Let } I = \int \frac{2x + 6}{\sqrt{x^2 + 6x + 3}} dx$$

$$\frac{d}{dx} (x^2 + 6x + 3) = 2x + 6$$

$$\therefore I = \int \frac{\frac{d}{dx} (x^2 + 6x + 3)}{\sqrt{x^2 + 6x + 3}} dx$$

$$\therefore I = 2\sqrt{x^2 + 6x + 3} + c \quad \dots \left[ \because \int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c \right]$$

Exercise 5.2 | Q 9 | Page 123

**Evaluate the following.**

$$\int \frac{1}{\sqrt{x} + x} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1}{\sqrt{x} + x} dx$$

$$= \int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$$

$$\text{Put } 1 + \sqrt{x} = t$$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore \frac{1}{\sqrt{x}} dx = 2 dt$$

$$\therefore I = \int \frac{2 \cdot dt}{t}$$

$$= 2 \int \frac{1}{t} \cdot dt$$

$$= 2 \log |t| + c$$

$$\therefore I = 2 \log |1 + \sqrt{x}| + c$$

**Exercise 5.2 | Q 10 | Page 123**

**Evaluate the following.**

$$\int \frac{1}{x(x^6 + 1)} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1}{x(x^6 + 1)} dx$$

$$= \frac{x^5}{x^6(x^6 + 1)} dx$$

$$\text{Put } x^6 = t$$

$$\therefore 6x^5 dx = dt$$

$$\therefore x^5 \cdot dx = \frac{1}{6} \cdot dt$$

$$\therefore I = \frac{1}{6} \int \frac{dt}{t(t+1)}$$

$$= \frac{1}{6} \int \frac{(t+1) - t}{t(t+1)} dt$$

$$= \frac{1}{6} \int \left( \frac{1}{t} - \frac{1}{t+1} \right) dt$$

$$= \frac{1}{6} [\log |t| - \log |t+1|] + c$$

$$= \frac{1}{6} \log \left| \frac{t}{t+1} \right| + c$$

$$\therefore I = \frac{1}{6} \log \left| \frac{x^6}{x^6 + 1} \right| + c$$

**EXERCISE 5.3 [PAGE 123]**

**Evaluate the following.**

$$\int \frac{(3e)^{2t} + 5}{4e^{2t} - 5} dt$$

**Solution:**

$$\text{Let } I = \int \frac{(3e)^{2t} + 5}{4e^{2t} - 5} dt$$

$$\text{Let } (3e)^{2t} + 5 = A(4e^{2t} - 5) + B \frac{d}{dt}(4e^{2t} - 5)$$

$$= 4Ae^{2t} - 5A + B(8e^{2t})$$

$$\therefore (3e)^{2t} + 5 = (4A + 8B)e^{2t} - 5A$$

Comparing the coefficients of  $e^{2t}$  and constant term on both sides, we get

$$4A + 8B = 3 \text{ and } -5A = 5$$

Solving these equations, we get

$$A = -1 \text{ and } B = \frac{7}{8}$$

$$\therefore I = \int \frac{-1(4e^{2t} - 5) + \frac{7}{8}(8e^{2t})}{4e^{2t} - 5} dt$$

$$= - \int dt + \frac{7}{8} \int \frac{8e^{2t}}{4e^{2t} - 5} dt$$

$$\therefore I = -t + \frac{7}{8} \log|4e^{2t} - 5| + c \quad \dots \left[ \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$



**Evaluate the following.**

$$\int \frac{20 - 12e^x}{3e^x - 4} dx$$

**Solution:**

$$\text{Let } I = \int \frac{20 - 12e^x}{3e^x - 4} dx$$

$$\text{Let } 20 - 12e^x = A(3e^x - 4) + B \frac{d}{dx}(3e^x - 4)$$

$$= 3Ae^x - 4A + 3Be^x$$

$$\therefore 20 - 12e^x = (3A + 3B)e^x - 4A$$

Comparing the coefficients of  $e^x$  and constant term on both sides, we get

$$-4A = 20 \text{ and } 3A + 3B = -12$$

Solving these equations, we get

$$A = -5 \text{ and } B = 1$$

$$\therefore I = \int \frac{-5(3e^x - 4) + 3e^x}{3e^x - 4} dx$$

$$= -5 \int dx + \int \frac{3e^x}{3e^x - 4} dx$$

$$\therefore I = -5x + \log |(3e^x - 4)| + c \quad \dots \left[ \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

**Exercise 5.3 | Q 3 | Page 123**

**Evaluate the following.**

$$\int \frac{3e^x + 4}{2e^x - 8} dx$$

**Solution:**

$$\text{Let } I = \int \frac{3e^x + 4}{2e^x - 8} dx$$

$$\text{Let } 3e^x + 4 = A(2e^x - 8) + B \frac{d}{dx}(2e^x - 8)$$

$$= 2Ae^x - 8A + B(2e^x)$$

$$\therefore 3e^x + 4 = (2A + 2B)e^x - 8A$$

Comparing the coefficients of  $e^x$  and constant term on both sides, we get

$$2A + 2B = 3 \text{ and } -8A = 4$$

Solving these equations, we get

$$A = -\frac{1}{2} \text{ and } B = 2$$

$$\therefore I = \int \frac{-\frac{1}{2}(2e^x - 8) + 2(2e^x)}{2e^x - 8} dx$$

$$= -\frac{1}{2} \int dx + 2 \int \frac{2e^x}{2e^x - 8} dx$$

$$\therefore I = -\frac{1}{2}x + 2 \log|2e^x - 8| + c \dots \left[ \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

**Exercise 5.3 | Q 4 | Page 123**

**Evaluate the following.**

$$\int \frac{2e^x + 5}{2e^x + 1} dx$$

**Solution:**

$$\text{Let } I = \int \frac{2e^x + 5}{2e^x + 1} dx$$

$$\text{Let } 2e^x + 5 = A(2e^x + 1) + B \frac{d}{dx}(2e^x + 1)$$

$$= 2Ae^x + A + B(2e^x)$$

$$\therefore 2e^x + 5 = (2A + 2B)e^x + A$$

Comparing the coefficients of  $e^x$  and constant term on both sides, we get

$$2A + 2B = 2 \text{ and } A = 5$$

Solving these equations, we get

$$B = -4$$

$$\therefore I = \int \frac{5(2e^x + 1) - 4(2e^x)}{2e^x + 1} dx$$

$$= 5 \int dx - 4 \int \frac{2e^x}{2e^x + 1} dx$$

$$\therefore I = 5x - 4 \log |2e^x + 1| + c \quad \dots \left[ \int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c \right]$$

#### EXERCISE 5.4 [PAGES 128 - 129]

##### Exercise 5.4 | Q 1 | Page 129

**Evaluate the following.**

$$\int \frac{1}{4x^2 - 1} dx$$

**Solution:**

$$\text{Let } I = \int \frac{dx}{4x^2 - 1}$$

$$= \frac{1}{4} \int \frac{dx}{x^2 - \frac{1}{4}}$$

$$\begin{aligned}
 &= \frac{1}{4} \int \frac{dx}{x^2 - \left(\frac{1}{2}\right)^2} \\
 &= \frac{1}{4} \times \frac{1}{2 \times \frac{1}{2}} \log \left| \frac{x - \frac{1}{2}}{x + \frac{1}{2}} \right| + c \\
 \therefore I &= \frac{1}{4} \log \left| \frac{2x - 1}{2x + 1} \right| + c
 \end{aligned}$$

**Alternate Method:**

$$\begin{aligned}
 \text{Let } I &= \int \frac{dx}{4x^2 - 1} = \int \frac{dx}{(2x)^2 - (1)^2} \\
 &= \frac{1}{2 \times 1} \times \frac{1}{2} \log \left| \frac{2x - 1}{2x + 1} \right| + c \\
 \therefore I &= \frac{1}{4} \log \left| \frac{2x - 1}{2x + 1} \right| + c
 \end{aligned}$$

**Exercise 5.4 | Q 2 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{x^2 + 4x - 5} dx$$

**Solution:**

$$\begin{aligned}
 \text{Let } I &= \int \frac{1}{x^2 + 4x - 5} dx \\
 &= \int \frac{1}{x^2 + 4x + 4 - 4 - 5} dx \\
 &= \int \frac{1}{(x + 2)^2 - 9} dx \\
 &= \int \frac{1}{(x + 2)^2 - 3^2} dx
 \end{aligned}$$

$$= \frac{1}{2 \times 3} \log \left| \frac{(x+2) - 3}{(x+2) + 3} \right| + c$$

$$\therefore I = \frac{1}{6} \log \left| \frac{x-1}{x+5} \right| + c$$

**Exercise 5.4 | Q 3 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{4x^2 - 20x + 17} dx$$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{4x^2 - 20x + 17} dx \\ &= \frac{1}{4} \int \frac{1}{x^2 - 5x + \frac{17}{4}} dx \\ &= \frac{1}{4} \int \frac{1}{x^2 - 2 \cdot \frac{5}{2}x + \frac{25}{4} - \frac{25}{4} + \frac{17}{4}} dx \\ &= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \frac{8}{4}} dx \\ &= \frac{1}{4} \int \frac{1}{\left(x - \frac{5}{2}\right)^2 - \left(\sqrt{2}\right)^2} + c \\ &= \frac{1}{4} \times \frac{1}{2\sqrt{2}} \log \left| \frac{x - \frac{5}{2} - \sqrt{2}}{x - \frac{5}{2} + \sqrt{2}} \right| + c \\ \therefore I &= \frac{1}{8\sqrt{2}} \log \left| \frac{2x - 5 - 2\sqrt{2}}{2x - 5 + 2\sqrt{2}} \right| + c \end{aligned}$$

Exercise 5.4 | Q 4 | Page 129

Evaluate the following.

$$\int \frac{x}{4x^2 - 2x^2 - 3} dx$$

Solution:

$$\text{Let } I = \int \frac{x}{4x^2 - 2x^2 - 3} dx$$

$$\text{Put } x^2 = t$$

$$\therefore 2x dx = dt$$

$$\therefore x dx = \frac{dt}{2}$$

$$\therefore I = \frac{1}{2} \int \frac{dt}{4t^2 - 2t - 3}$$

$$= \frac{1}{2 \times 4} \int \frac{dt}{t^2 - \frac{1}{2}t - \frac{3}{4}}$$

$$= \frac{1}{8} \int \frac{dt}{t^2 - \frac{1}{2}t - \frac{3}{4}}$$

$$= \frac{1}{8} \int \frac{dt}{t^2 - 2 \cdot \frac{1}{4} \cdot t + \frac{1}{16} - \frac{1}{16} - \frac{3}{4}}$$

$$= \frac{1}{8} \int \frac{dt}{t^2 - 2 \cdot \frac{1}{4} \cdot t + \frac{1}{16} - \frac{1}{16} - \frac{3}{4}}$$

$$= \frac{1}{8} \int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \left(\frac{1+12}{16}\right)}$$

$$= \frac{1}{8} \int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \frac{13}{16}}$$

$$\begin{aligned}
&= \frac{1}{8} \int \frac{dt}{\left(t - \frac{1}{4}\right)^2 - \left(\frac{\sqrt{13}}{4}\right)^2} \\
&= \frac{1}{8} \cdot \frac{1}{2 \times \frac{\sqrt{13}}{4}} \log \left| \frac{t - \frac{1}{4} - \frac{\sqrt{13}}{4}}{t - \frac{1}{4} + \frac{\sqrt{13}}{4}} \right| + c \\
&= \frac{1}{4\sqrt{13}} \log \left| \frac{4t - 1 - \sqrt{13}}{4t - 1 + \sqrt{13}} \right| + c \\
\therefore I &= \frac{1}{4\sqrt{13}} \log \left| \frac{4x^2 - 1 - \sqrt{13}}{4x^2 - 1 + \sqrt{13}} \right| + c
\end{aligned}$$

**Exercise 5.4 | Q 5 | Page 128**

**Evaluate the following.**

$$\int \frac{x^3}{16x^8 - 25} dx$$

**Solution:**

$$\text{Let } I = \int \frac{x^3}{16x^8 - 25} dx$$

$$\text{Put } x^4 = t$$

$$\therefore 4x^3 dx = dt$$

$$\therefore x^3 dx = \frac{1}{4} dt$$

$$\therefore I = \frac{1}{4} \int \frac{dt}{16t^2 - 25}$$

$$= \frac{1}{4 \times 16} \int \frac{dt}{t^2 - \frac{25}{16}}$$

$$\begin{aligned}
&= \frac{1}{64} \int \frac{dt}{t^2 - \left(\frac{5}{4}\right)^2} \\
&= \frac{1}{64} \times \frac{1}{2 \times \frac{5}{4}} \log \left| \frac{t - \frac{5}{4}}{t + \frac{5}{4}} \right| + c \\
&= \frac{1}{160} \log \left| \frac{4t - 5}{4t + 5} \right| + xc \\
\therefore I &= \frac{1}{160} \log \left| \frac{4x^4 - 5}{4x^4 + 5} \right| + c
\end{aligned}$$

**Exercise 5.4 | Q 6 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{a^2 - b^2 x^2} dx$$

**Solution:**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{a^2 - b^2 x^2} dx \\
&= \frac{1}{b^2} \int \frac{1}{\frac{a^2}{b^2} - x^2} dx \\
&= \frac{1}{b^2} \int \frac{1}{\left(\frac{a}{b}\right)^2 - x^2} dx \\
&= \frac{1}{b^2} \times \frac{1}{2\left(\frac{a}{b}\right)} \log \left| \frac{\frac{a}{b} + x}{\frac{a}{b} - x} \right| + c \\
\therefore I &= \frac{1}{2ab} \log \left| \frac{a + bx}{a - bx} \right| + c
\end{aligned}$$

**Alternate Method:**

$$\text{Let } I = \int \frac{dx}{a^2 - b^2 x^2} = \int \frac{dx}{a^2 - (bx)^2}$$



$$= \frac{1}{2 \times a} \times \frac{1}{b} \log \left| \frac{a + bx}{a - bx} \right| + c$$

$$\therefore I = \frac{1}{2ab} \log \left| \frac{a + bx}{a - bx} \right| + c$$

**Exercise 5.4 | Q 7 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{7 + 6x - x^2} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1}{7 + 6x - x^2} dx$$

$$= \int \frac{1}{7 + 9 - 9 + 6x - x^2} dx$$

$$= \int \frac{1}{16 - (x^2 - 6x + 9)} dx$$

$$= \int \frac{1}{(4)^2 - (x - 3)^2} dx$$

$$= \frac{1}{2 \times 4} \log \left| \frac{4 + x - 3}{4 - (x - 3)} \right| + c$$

$$\therefore I = \frac{1}{8} \log \left| \frac{1 + x}{7 - x} \right| + c$$

**Exercise 5.4 | Q 8 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{\sqrt{3x^2 + 8}} dx$$

**Solution:**

$$\text{Let } I = \int \frac{1}{\sqrt{3x^2 + 8}} dx$$

$$\int \frac{1}{\sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2}} dx$$

$$= \frac{\log \left| \sqrt{3}x + \sqrt{(\sqrt{3}x)^2 + (\sqrt{8})^2} \right|}{\sqrt{3}} + c$$

$$\therefore I = \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 + 8} \right| + c$$

**Alternate method:**

$$\text{Let } I = I = \int \frac{1}{\sqrt{3x^2 + 8}} dx = \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \frac{8}{3}}} dx$$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2}} dx$$

$$= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \left(\frac{2\sqrt{2}}{\sqrt{3}}\right)^2} \right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 + \frac{8}{3}} \right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3}x + \sqrt{3x^2 + 8}}{\sqrt{3}} \right| + c_1$$

$$= \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 + 8} \right| - \frac{1}{\sqrt{3}} \log \sqrt{3} + c_1$$

$$\therefore I = \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 + 8} \right| + c$$

$$\text{where } c = c_1 - \frac{1}{\sqrt{3}} \log \sqrt{3}$$

**Exercise 5.4 | Q 9 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{\sqrt{x^2 + 4x + 29}} dx$$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{\sqrt{x^2 + 4x + 29}} dx \\ &= \int \frac{1}{\sqrt{x^2 + 2 \cdot 2x + 4 - 4 + 29}} dx \\ &= \int \frac{1}{\sqrt{(x + 2)^2 + 25}} dx \\ &= \int \frac{dx}{\sqrt{(x + 2)^2 + 5^2}} \\ &= \log \left| (x + 2) + \sqrt{(x + 2)^2 + 5^2} \right| + c \\ \therefore I &= \log \left| (x + 2) + \sqrt{x^2 + 4x + 29} \right| + c \end{aligned}$$

**Exercise 5.4 | Q 10 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{\sqrt{3x^2 - 5}} dx$$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{1}{\sqrt{3x^2 - 5}} dx \\&= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 - \frac{5}{3}}} dx \\&= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{x^2 - \left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2}} dx \\&= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 - \left(\frac{\sqrt{5}}{\sqrt{3}}\right)^2} \right| + c_1 \\&= \frac{1}{\sqrt{3}} \log \left| x + \sqrt{x^2 - \frac{5}{3}} \right| + c_1 \\&= \frac{1}{\sqrt{3}} \log \left| \frac{\sqrt{3}x + \sqrt{3x^2 - 5}}{\sqrt{3}} \right| + c_1 \\&= \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 - 5} \right| - \frac{1}{\sqrt{3}} \log \sqrt{3} + c_1 \\&\therefore I = \frac{1}{\sqrt{3}} \log \left| \sqrt{3}x + \sqrt{3x^2 - 5} \right| + c, \\&\text{where } c = c_1 - \frac{1}{\sqrt{3}} \log \sqrt{3}\end{aligned}$$

**Exercise 5.4 | Q 11 | Page 129**

**Evaluate the following.**

$$\int \frac{1}{\sqrt{x^2 - 8x - 20}} dx$$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{1}{\sqrt{x^2 - 8x - 20}} dx \\&= \int \frac{1}{\sqrt{x^2 - 2 \cdot 4x + 16 - 16 - 20}} dx \\&= \int \frac{dx}{\sqrt{(x-4)^2 - 36}} dx \\&= \int \frac{dx}{\sqrt{(x-4)^2 - 6^2}} dx \\&= \log \left| (x-4) + \sqrt{(x-4)^2 - 6^2} \right| + c \\ \therefore I &= \log \left| (x-4) + \sqrt{x^2 - 8x - 20} \right| + c\end{aligned}$$

#### EXERCISE 5.5 [PAGE 133]

##### Exercise 5.5 | Q 1 | Page 133

Evaluate the following.

$$\int x \log x \, dx$$

**Solution:** Let  $I = \int x \log x \, dx$

$$\begin{aligned}&= \log x \int x \, dx - \int \left[ \frac{d}{dx} (\log x) \int x \, dx \right] dx \\&= \log x \cdot \frac{x^2}{2} - \int \left[ \frac{1}{x} \times \frac{x^2}{2} \right] dx \\&= \frac{x^2}{2} \log x - \frac{1}{2} \int x \, dx\end{aligned}$$

$$= \frac{x^2}{2} \log x - \frac{1}{2} \cdot \frac{x^2}{2} + c$$

$$\therefore I = \frac{x^2}{2} \log x - \frac{x^2}{4} + c$$

**Exercise 5.5 | Q 2 | Page 133**

**Evaluate the following.**

$$\int x^2 e^{4x} dx$$

**Solution:**

$$\text{Let } I = \int x^2 e^{4x} dx$$

$$= x^2 \int e^{4x} dx - \int \left[ \frac{d}{dx} (x^2) \int e^{4x} dx \right] dx$$

$$= x^2 \cdot \frac{e^{4x}}{4} - \int 2x \cdot \frac{e^{4x}}{4} dx$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2} \int x \cdot e^{4x} dx$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2} \left[ x \int e^{4x} dx - \int \left( \frac{d}{dx} (x) \int e^{4x} dx \right) dx \right]$$

$$= \frac{x^2 \cdot e^{4x}}{4} - \frac{1}{2} \left[ x \cdot \frac{e^{4x}}{4} - \int 1 \cdot \frac{e^{4x}}{4} dx \right]$$

$$= \frac{x^2 e^{4x}}{4} - \frac{1}{2} \left[ \frac{x \cdot e^{4x}}{4} - \frac{1}{4} \int e^{4x} dx \right]$$

$$= \frac{x^2 e^{4x}}{4} - \frac{1}{2} \left[ \frac{x \cdot e^{4x}}{4} - \frac{1}{4} \cdot \frac{e^{4x}}{4} \right] + c$$

$$= \frac{x^2 e^{4x}}{4} - \frac{x e^{4x}}{8} + \frac{e^{4x}}{32} + c$$

$$\therefore I = \frac{e^{4x}}{4} \left[ x^2 - \frac{x}{2} + \frac{1}{8} \right] + c$$

**Exercise 5.5 | Q 3 | Page 133**

**Evaluate the following.**

$$\int x^2 e^{3x} dx$$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int x^2 e^{3x} dx \\ &= x^2 \int e^{3x} dx - \int \left[ \frac{d}{dx} (x^2) \int e^{3x} dx \right] dx \\ &= x^2 \cdot \left( \frac{e^{3x}}{3} \right) - \int 2x \cdot \frac{e^{3x}}{3} dx \\ &= \frac{x^2}{3} e^{3x} - \frac{2}{3} \int x \cdot e^{3x} dx \\ &= \frac{x^2}{3} e^{3x} - \frac{2}{3} \left[ x \int e^{3x} dx - \int \left( \frac{d}{dx} (x) \int e^{3x} dx \right) dx \right] \\ &= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3} \left[ x \cdot \frac{e^{3x}}{3} - \int 1 \cdot \frac{e^{3x}}{3} dx \right] \\ &= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3} \left[ \frac{1}{3} x e^{3x} - \frac{1}{3} \int e^{3x} dx \right] \\ &= \frac{x^2 \cdot e^{3x}}{3} - \frac{2}{3} \left[ \frac{1}{3} x e^{3x} - \frac{1}{3} \cdot \frac{e^{3x}}{3} \right] + c \\ \therefore I &= \frac{1}{3} x^2 \cdot e^{3x} - \frac{2}{9} x e^{3x} + \frac{2}{27} e^{3x} + c \end{aligned}$$

Exercise 5.5 | Q 4 | Page 133

Evaluate the following.

$$\int x^3 e^{x^2} dx$$

**Solution:**

$$\text{Let } I = \int x^3 e^{x^2} dx$$

$$= \int x^2 \cdot x \cdot e^{x^2} dx$$

$$\text{Put } x^2 = t$$

$$\therefore 2x \cdot dx = dt$$

$$\therefore x dx = \frac{dt}{2}$$

$$\therefore I = \frac{1}{2} \int t e^t dt$$

$$= \frac{1}{2} \left[ t \int e^t dt - \int \left[ \frac{d}{dt}(t) \int e^t dt \right] dt \right]$$

$$= \frac{1}{2} \left[ t e^t - \int 1 \cdot e^t dt \right]$$

$$= \frac{1}{2} (t e^t - e^t) + c = \frac{1}{2} e^t (t - 1) + c$$

$$\therefore I = \frac{1}{2} e^{x^2} (x^2 - 1) + c$$

Exercise 5.5 | Q 5 | Page 133

Evaluate the following.

$$\int e^x \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$$



**Solution:**

$$\text{Let } I = \int e^x \left( \frac{1}{x} - \frac{1}{x^2} \right) dx$$

$$\text{Put } f(x) = \frac{1}{x}$$

$$\therefore f'(x) = \frac{1}{x^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$\therefore I = e^x \cdot \frac{1}{x} + c$$

**Exercise 5.5 | Q 6 | Page 133**

**Evaluate the following.**

$$\int e^x \frac{x}{(x+1)^2} dx$$

**Solution:**

$$\text{Let } I = \int \left( \frac{x}{(x+1)^2} \right) e^x dx$$

$$= \int e^x \left( \frac{(x+1) - 1}{(x+1)^2} \right) dx$$

$$= \int e^x \left( \frac{x+1}{(x+1)^2} - \frac{1}{(x+1)^2} \right) dx$$

$$= \int e^x \left( \frac{1}{x+1} - \frac{1}{(x+1)^2} \right) dx$$

$$\text{Put } f(x) = \frac{1}{x+1}$$

$$\therefore f'(x) = \frac{-1}{(x+1)^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$\therefore I = e^x \left( \frac{1}{x+1} \right) + c$$

**Exercise 5.5 | Q 7 | Page 133**

**Evaluate the following.**

$$\int e^x \frac{x-1}{(x+1)^3} dx$$

**Solution:**

$$\text{Let } I = \int e^x \frac{(x-1)}{(x+1)^3} dx = \int e^x \frac{(x+1-1-1)}{(x+1)^3} dx$$

$$= \int e^x \left[ \frac{x+1}{(x+1)^3} - \frac{2}{(x+1)^3} \right] dx$$

$$= \int e^x \left[ \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] dx$$

$$\text{Put } f(x) = \frac{1}{(x+1)^2}$$

$$\therefore f'(x) = \frac{-2}{(x+1)^3}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x f(x) + c$$

$$= e^x \times \frac{1}{(x+1)^2} + c$$

$$\therefore I = \frac{e^x}{(x+1)^2} + c$$

**Exercise 5.5 | Q 8 | Page 133**

**Evaluate the following.**

$$\int e^x \left[ (\log x)^2 + \frac{2 \log x}{x} \right] dx$$

**Solution:**

$$\text{Let } I = \int e^x \left[ (\log x)^2 + \frac{2 \log x}{x} \right] dx$$

$$\text{Put } f(x) = (\log x)^2$$

$$\therefore f'(x) = \frac{2 \log x}{x}$$

$$\therefore I = \int e^x [f(x) + f'(x)] + dx$$

$$= e^x f(x) + c$$

$$\therefore I = e^x (\log x)^2 + c$$

**Exercise 5.5 | Q 9 | Page 133**

**Evaluate the following.**

$$\int \left[ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$$

**Solution:**

$$\text{Let } I = \int \left[ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$$

$$\text{Put } \log x = t$$

$$\therefore x = e^t$$

$$\therefore dx = e^t dt$$

$$\therefore I = \int e^t \left[ \frac{1}{t} - \frac{1}{t^2} \right] dt$$

$$\text{Put } f(t) = \frac{1}{t}$$

$$\therefore f'(x) = \frac{-1}{t^2}$$

$$\therefore I = \int e^t [f(t) + f'(x)] dt$$

$$= e^t f(t) + c$$

$$\therefore I = e^t \left( \frac{1}{t} \right) + c = \frac{x}{\log x} + c$$

**Exercise 5.5 | Q 10 | Page 133**

**Evaluate the following.**

$$\int \frac{\log x}{(1 + \log x)^2} dx$$

**Solution:**

$$\text{Let } I = \int \frac{\log x}{(1 + \log x)^2} dx$$

$$\text{Put } \log x = t$$

$$\therefore x = e^t$$

$$\therefore dx = e^t dt$$

$$\therefore I = \int \frac{t}{(1+t)^2} e^t dt$$

$$= \int e^t \left[ \frac{(t+1) - 1}{(1+t)^2} \right] dt$$

$$= \int e^t \left[ \frac{t+1}{(1+t)^2} - \frac{1}{(1+t)^2} \right] dt$$

$$= \int e^t \left[ \frac{1}{(1+t)^2} - \frac{1}{(1+t)^2} \right] dt$$

$$\text{Put } f(t) = \frac{1}{1+t}$$

$$\therefore f'(t) = \frac{-1}{(1+t)^2}$$

$$\therefore \int e^t [f(t) + f'(t)] dt$$

$$= e^t f(t) + c$$

$$= e^t \cdot \frac{1}{1+t} + c$$

$$\therefore I = \frac{x}{1 + \log x} + c$$

## EXERCISE 5.6 [PAGE 135]

### Exercise 5.6 | Q 1 | Page 135

**Evaluate:**  $\int \frac{2x + 1}{(x + 1)(x - 2)} dx$

**Solution:**

Let  $I = \int \frac{2x + 1}{(x + 1)(x - 2)} dx$

Let  $\frac{2x + 1}{(x + 1)(x - 2)} = \frac{A}{x + 1} + \frac{B}{x - 2}$

$\therefore 2x + 1 = A(x - 2) + B(x + 1) \dots(i)$

Putting  $x = -1$  in (i), we get

$2(-1) + 1 = A(-3) + B(0)$

$\therefore -1 = -3A$

$\therefore A = \frac{1}{3}$

Putting  $x = 2$  in (i), we get

$2(2) + 1 = A(0) + B(3)$

$\therefore 5 = 3B$

$\therefore B = \frac{5}{3}$

$\therefore \frac{2x + 1}{(x + 1)(x - 2)} = \frac{\frac{1}{3}}{x + 1} + \frac{\frac{5}{3}}{x - 2}$

$\therefore I = \int \left( \frac{\left(\frac{1}{3}\right)}{x + 1} + \frac{\left(\frac{5}{3}\right)}{x - 2} \right) dx$

$$\therefore \frac{1}{3} \int \frac{1}{x+1} dx + \frac{5}{3} \int \frac{1}{x-2} dx$$

$$\therefore I = \frac{1}{3} \log|x+1| + \frac{5}{3} \log|x-2| + c$$

**Exercise 5.6 | Q 2 | Page 135**

**Evaluate:**  $\int \frac{2x+1}{x(x-1)(x-4)} dx$

**Solution:**

$$\text{Let } I = \int \frac{2x+1}{x(x-1)(x-4)} dx$$

$$\text{Let } \frac{2x+1}{x(x-1)(x-4)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x-4}$$

$$\therefore 2x+1 = A(x-1)(x-4) + Bx(x-4) + Cx(x-1) \dots(i)$$

Putting  $x = 0$  in (i), we get

$$0+1 = A(0-1)(0-4) + B(0)(-4) + C(0)(-1)$$

$$\therefore 1 = 4A$$

$$\therefore A = \frac{1}{4}$$

Putting  $x = 1$  in (i), we get

$$2(1)+1 = A(0)(-3) + B(1)(1-4) + C(1)(0)$$

$$\therefore 3 = -3B$$

$$\therefore B = -1$$

Putting  $x = 4$  in (i), we get

$$2(4)+1 = A(3)(0) + B(4)(0) + C(4)(4-1)$$

$$\therefore 9 = C(4)(3)$$

$$\therefore C = \frac{3}{4}$$

$$\therefore \frac{2x+1}{x(x-1)(x-4)} = \frac{\frac{1}{4}}{x} + \frac{-1}{x-1} + \frac{\frac{3}{4}}{x-4}$$

$$\begin{aligned}\therefore I &= \int \left( \frac{\frac{1}{4}}{x} + \frac{-1}{x-1} + \frac{\frac{3}{4}}{x-4} \right) dx \\ &= \frac{1}{4} \int \frac{1}{x} dx - \int \frac{1}{x-1} dx + \frac{3}{4} \int \frac{1}{x-4} dx \\ \therefore I &= \frac{1}{4} \log|x| - \log|x-1| + \frac{3}{4} \log|x-4| + c\end{aligned}$$

#### Exercise 5.6 | Q 3 | Page 135

**Evaluate:**  $\int \frac{x^2 + x - 1}{x^2 + x - 6} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \frac{x^2 + x - 1}{x^2 + x - 6} dx \\ &= \int \frac{(x^2 + x - 6) + 5}{x^2 + x - 6} dx \\ &= \int \left[ \frac{x^2 + x - 6}{x^2 + x - 6} + \frac{5}{x^2 + x - 6} \right] dx \\ &= \int \left[ 1 + \frac{5}{x^2 + x - 6} \right] dx \\ &= \int \left[ 1 + \frac{5}{(x+3)(x-2)} \right] dx \\ \text{Let } \frac{5}{(x+3)(x-2)} &= \frac{A}{x+3} + \frac{B}{x-2}\end{aligned}$$

$$\therefore 5 = A(x-2) + B(x+2) \dots(i)$$



Putting  $x = 2$  in (i), we get

$$5 = A(0) + B(5)$$

$$\therefore 5 = 5B$$

$$\therefore B = 1$$

Putting  $x = -3$  in (i), we get

$$5 = A(-5) + B(0)$$

$$\therefore 6 = -5A$$

$$\therefore A = -1$$

$$\therefore \frac{5}{(x+3)(x-2)} = \frac{-1}{x+3} + \frac{1}{x-2}$$

$$\begin{aligned}\therefore I &= \int \left[ 1 + \frac{-1}{x+3} + \frac{1}{x-2} \right] dx \\ &= \int dx - \int \frac{1}{x+3} dx + \int \frac{1}{x-2} dx\end{aligned}$$

$$\therefore I = x - \log |x+3| + \log |x-2| + c$$

#### Exercise 5.6 | Q 4 | Page 135

**Evaluate:**  $\int \frac{x}{(x-1)^2(x+2)} dx$

**Solution:**

$$\text{Let } I = \int \frac{x}{(x-1)^2(x+2)} dx$$

$$\text{Let } \frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$$

$$\therefore x = A(x-1)(x+2) + B(x+2) + C(x-1)^2 \dots (i)$$

Putting  $x = 1$  in (i), we get

$$1 = A(0)(3) + B(3) + C(0)^2$$

$$\therefore 1 = 3B$$

$$\therefore B = 1/3$$

Putting  $x = -2$  in (i), we get

$$-2 = A(-3)(0) + B(0) + C(9)$$

$$\therefore -2 = 9C$$

$$\therefore C = -2/9$$

Putting  $x = -1$  in (i), we get

$$-1 = A(-2)(1) + B(1) + C(4)$$

$$\therefore -1 = -2A + \frac{1}{3} - \frac{8}{9}$$

$$\therefore -1 = -2A - \frac{5}{9}$$

$$\therefore 2A = -\frac{5}{9} + 1 = \frac{4}{9}$$

$$\therefore A = \frac{2}{9}$$

$$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2}$$

$$\begin{aligned} \therefore I &= \int \left[ \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{-\frac{2}{9}}{x+2} \right] dx \\ &= \frac{2}{9} \int \frac{1}{x-1} dx + \frac{1}{3} \int (x-1)^{-2} dx - \frac{2}{9} \int \frac{1}{x+2} dx \\ &= \frac{2}{9} \log|x-1| + \frac{1}{3} \cdot \frac{(x-1)^{-1}}{-1} - \frac{2}{9} \log|x+2| + c \\ &= \frac{2}{9} \log|x-1| - \frac{2}{9} \log|x+2| - \frac{1}{3} \times \frac{1}{x-1} + c \\ \therefore I &= \frac{2}{9} \log \left| \frac{x-1}{x+2} \right| - \frac{1}{3(x-1)} + c \end{aligned}$$

**Exercise 5.6 | Q 5 | Page 135**

**Evaluate:**  $\int \frac{3x - 2}{(x + 1)^2(x + 3)} dx$

**Solution:**

Let  $I = \int \frac{3x - 2}{(x + 1)^2(x + 3)} dx$

Let  $\frac{3x - 2}{(x + 1)^2(x + 3)} = \frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{C}{x + 3}$

$$\therefore 3x - 2 = (x + 3) [A(x + 1) + B] + C(x + 1)^2 \dots(i)$$

Putting  $x = -1$  in (i), we get

$$3(-1) - 2 = (-1 + 3)[A(0) + B] + C(0)$$

$$\therefore -5 = 2B$$

$$\therefore B = -\frac{5}{2}$$

Putting  $x = -3$  in (i), we get

$$3(-3) - 2 = 0[A(-3 + 1) + B] + C(-2)^2$$

$$\therefore -11 = 4C$$

$$\therefore C = -\frac{11}{4}$$

Putting  $x = 0$  in (i), we get

$$3(0) - 2 = 3[A(0 + 1) + B] + C(0 + 1)^2$$

$$\therefore -2 = 3A + 3B + C$$

$$\therefore -2 = 3A + 3\left(-\frac{5}{2}\right) - \frac{11}{4}$$

$$\therefore 3A = -2 + \frac{15}{2} + \frac{11}{4} = \frac{-8 + 30 + 11}{4} = \frac{33}{4}$$

$$\therefore A = \frac{33}{4} \times \frac{1}{3} = \frac{11}{4}$$

$$\therefore \frac{3x-2}{(x+1)^2(x+3)} = \frac{\frac{11}{4}}{x+1} + \frac{-\frac{5}{2}}{(x+1)^2} + \frac{-\frac{11}{4}}{x+3}$$

$$\begin{aligned} \therefore I &= \int \left( \frac{\frac{11}{4}}{x+1} - \frac{\frac{5}{2}}{(x+1)^2} - \frac{\frac{11}{4}}{x+3} \right) dx \\ &= \frac{11}{4} \int \frac{dx}{x+1} - \frac{5}{2} \int (x+1)^{-2} dx - \frac{11}{4} \int \frac{dx}{x+3} \\ &= \frac{11}{4} \log|x+1| - \frac{5}{2} \left( -\frac{1}{x+1} \right) - \frac{11}{4} \log|x+3| + c \\ &= \frac{11}{4} [\log|x+1| - \log|x+3|] + \frac{5}{2(x+1)} + c \\ \therefore I &= \frac{11}{4} \log \left| \frac{x+1}{x+3} \right| + \frac{5}{2(x+1)} + c \end{aligned}$$

**Exercise 5.6 | Q 6 | Page 135**

**Evaluate:**  $\int \frac{1}{x(x^5+1)} dx$

**Solution:**

$$\text{Let } I = \int \frac{1}{x(x^5 + 1)} dx$$

$$\therefore I = \int \frac{x^4}{x^5(x^5 + 1)} dx$$

$$\text{Put } x^5 = t$$

$$\therefore 5x^4 dx = dt$$

$$\therefore x^4 dx = \frac{dt}{5}$$

$$\therefore I = \int \frac{1}{t(t + 1)} \cdot \frac{dt}{5}$$

$$\text{Let } \frac{1}{t(t + 1)} = \frac{A}{t} + \frac{B}{t + 1}$$

$$\therefore 1 = A(t + 1) + Bt \quad \dots(i)$$

Putting  $t = -1$  in (i), we get

$$1 = A(0) + B(-1)$$

$$\therefore 1 = -B$$

$$\therefore B = -1$$

Putting  $t = 0$  in (i), we get

$$1 = A(1) + B(0)$$

$$\therefore A = 1$$

$$\therefore \frac{1}{t(t + 1)} = \frac{1}{t} + \frac{-1}{t + 1}$$

$$\therefore I = \frac{1}{5} \int \left( \frac{1}{t} + \frac{-1}{t + 1} \right) dt$$

$$\begin{aligned}
&= \frac{1}{5} \left[ \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right] \\
&= \frac{1}{5} [\log|t| - \log|t+1|] + c \\
&= \frac{1}{5} \log \left| \frac{t}{t+1} \right| + c \\
\therefore I &= \frac{1}{5} \log \left| \frac{x^5}{x^5+1} \right| + c
\end{aligned}$$

**Exercise 5.6 | Q 7 | Page 135**

**Evaluate:**  $\int \frac{1}{x(x^n+1)} dx$

**Solution:**

$$\begin{aligned}
\text{Let } I &= \int \frac{1}{x(x^n+1)} dx \\
\therefore I &= \int \frac{x^{n-1}}{x^{n-1} \times x(x^n+1)} dx \\
\therefore I &= \int \frac{x^{n-1}}{x^n(x^n+1)} dx
\end{aligned}$$

Put  $x^n = t$

$$\therefore nx^{n-1} dx = dt$$

$$\therefore x^{n-1} dx = \frac{dt}{n}$$

$$\therefore I = \int \frac{1}{t(t+1)} \cdot \frac{dt}{n}$$

$$\text{Let } \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1}$$

$$\therefore 1 = A(t + 1) + Bt \quad \dots(i)$$

Putting  $t = -1$  in (i), we get

$$1 = A(0) + B(-1)$$

$$\therefore 1 = -B$$

$$\therefore B = -1$$

Putting  $t = 0$  in (i), we get

$$1 = A(1) + B(0)$$

$$\therefore A = 1$$

$$\therefore \frac{1}{t(t+1)} = \frac{1}{t} + \frac{-1}{t+1}$$

$$\therefore I = \frac{1}{n} \int \left( \frac{1}{t} + \frac{-1}{t+1} \right) dt$$

$$= \frac{1}{n} \left[ \int \frac{1}{t} dt - \int \frac{1}{t+1} dt \right]$$

$$= \frac{1}{n} [\log|t| - \log|t+1|] + c$$

$$= \frac{1}{n} \log \left| \frac{t}{t+1} \right| + c$$

$$\therefore I = \frac{1}{n} \log \left| \frac{x^n}{x^n + 1} \right| + c$$

**Exercise 5.6 | Q 8 | Page 135**

**Evaluate:**  $\int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

**Solution:**

Let  $I = \int \frac{5x^2 + 20x + 6}{x^3 + 2x^2 + x} dx$

$$= \int \frac{5x^2 + 20x + 6}{x(x^2 + 2x + 1)} dx$$

$$= \int \frac{5x^2 + 20x + 6}{x(x + 1)^2} dx$$

$$\text{Let } \frac{5x^2 + 20x + 6}{x(x + 1)^2} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{(x + 1)^2}$$

$$\therefore 5x^2 + 20x + 6 = A(x + 1)^2 + B(x + 1)x + Cx \quad \dots(i)$$

Putting  $x = 0$  in (i), we get

$$5(0) + 20(0) + 6 = A(1)^2 + B(1)(0) + C(0)$$

$$\therefore A = 6$$

Putting  $x = -1$  in (i), we get

$$5(1) + 20(-1) + 6 = A(0) + B(0)(-1) + C(-1)$$

$$\therefore -9 = -C$$

$$\therefore C = 9$$

Putting  $x = 1$  in (i), we get

$$5(1) + 20(1) + 6 = A(2)^2 + B(2)(1) + C(1)$$

$$\therefore 31 = 4A + 2B + C$$

$$\therefore 31 = 4(6) + 2B + 9$$

$$\therefore B = -1$$

$$\therefore \frac{5x^2 + 20x + 6}{x(x + 1)^2} = \frac{6}{x} + \frac{-1}{x + 1} + \frac{9}{(x + 1)^2}$$

$$\therefore I = \int \left[ \frac{6}{x} + \frac{-1}{x + 1} + \frac{9}{(x + 1)^2} \right] dx$$



$$\begin{aligned}
 &= 6 \int \frac{1}{x} dx - \int \frac{1}{x+1} dx + 9 \int (x+1)^{-2} dx \\
 &= 6 \log|x| - \log|x+1| + 9 \frac{(x+1)^{-1}}{-1} + c \\
 \therefore I &= 6 \log|x| - \log|x+1| - \frac{9}{x+1} + c
 \end{aligned}$$

#### MISCELLANEOUS EXERCISE 5 [PAGES 137 - 139]

##### Miscellaneous Exercise 5 | Q 1.01 | Page 137

**Choose the correct alternative from the following.**

The value of  $\int \frac{dx}{\sqrt{1-x}}$  is

Options

$$2\sqrt{1-x} + c$$

$$- 2\sqrt{1-x} + c$$

$$\sqrt{x} + c$$

$$x + c$$

**Solution:**

$$- 2\sqrt{1-x} + c$$

##### Miscellaneous Exercise 5 | Q 1.02 | Page 137

**Choose the correct alternative from the following.**

$$\int \sqrt{1-x^2} dx =$$

Options

$$\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x + \sqrt{1+x^2}) + c$$

$$\frac{2}{3} (1+x^2)^{\frac{3}{2}} + c$$

$$\frac{1}{3} (1+x^2) + c$$

$$\frac{x}{\sqrt{1+x^2}} + c$$

**Solution:**

$$\frac{x}{2} \sqrt{1+x^2} + \frac{1}{2} \log(x + \sqrt{1+x^2}) + c$$

**Miscellaneous Exercise 5 | Q 1.03 | Page 137**

**Choose the correct alternative from the following.**

$$\int x^2 (3)^{x^3} dx =$$

Options

$$(3)^{x^3} + c$$

$$\frac{(3)^{x^3}}{3 \cdot \log 3} + c$$

$$\log 3 (3)^{x^3} + c$$

$$x^2 (3)^{x^3} + c$$

**Solution:**

$$\frac{(3)^{x^3}}{3 \cdot \log 3} + c$$

**Explanation:**

$$\text{Let } I = \int x^2 \cdot (3)^{x^3} dx$$

$$\text{Put } x^3 = t$$

$$\therefore 3x^2 dx = dt$$

$$\therefore x^2 dx = \frac{1}{3} dt$$

$$\therefore I = \frac{1}{3} \int 3^t \cdot dt$$

$$= \frac{1}{3} \cdot \frac{3^t}{\log 3} + c$$

$$= \frac{(3)^{x^3}}{3 \log 3} + c$$

**Miscellaneous Exercise 5 | Q 1.04 | Page 137**

Choose the correct alternative from the following.

$$\int \frac{x+2}{2x^2+6x+5} dx = p \int \frac{4x+6}{2x^2+6x+5} dx + \frac{1}{2} \int \frac{dx}{2x^2+6x+5}, \text{ then } p = ?$$

1.  $1/3$
2.  $1/2$
3.  $1/4$
4.  $2$

**Solution:**  $1/4$

**Explanation:**

$$\text{Let } x+2 = p \frac{d}{dx} (2x^2+6x+5) + q$$

$$= p(4x+6) + q$$

$$\therefore x+2 = 4px + 6p + q$$



$$\therefore 4p = 1 \text{ and } 6p + q = 2$$

$$\therefore p = 1/4$$

### Miscellaneous Exercise 5 | Q 1.05 | Page 137

**Choose the correct alternative from the following.**

$$\int \frac{dx}{(x - x^2)} =$$

1.  $\log x - \log (1 - x) + c$
2.  $\log (1 - x^2) + c$
3.  $-\log x + \log(1 - x) + c$
4.  $\log (x - x^2) + c$

**Solution:**  $\log x - \log (1 - x) + c$

**Explanation:**

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{(x - x^2)} \\ &= \int \frac{1}{x(1 - x)} dx \\ &= \int \frac{(1 - x) + x}{x(1 - x)} dx \\ &= \int \left( \frac{1}{x} + \frac{1}{1 - x} \right) dx \\ &= \log|x| + \frac{\log|1 - x|}{-1} + c \\ &= \log|x| - \log|1 - x| + c \end{aligned}$$

### Miscellaneous Exercise 5 | Q 1.06 | Page 137

**Choose the correct alternative from the following.**

$$\int \frac{dx}{(x - 8)(x + 7)} =$$

Options

$$\frac{1}{15} \log \left| \frac{x+2}{x-1} \right| + c$$

$$\frac{1}{15} \log \left| \frac{x+8}{x+7} \right| + c$$

$$\frac{1}{15} \log \left| \frac{x-8}{x+7} \right| + c$$

$$(x-8)(x-7) + c$$

**Solution:**

$$\frac{1}{15} \log \left| \frac{x-8}{x+7} \right| + c$$

**Explanation:**

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{(x-8)(x+7)} \\ &= \frac{1}{15} \int \frac{15 \cdot dx}{(x-8)(x+7)} \\ &= \frac{1}{15} \int \frac{(x+7) - (x-8)}{(x-8)(x+7)} dx \\ &= \frac{1}{15} \left( \int \frac{1}{x-8} - \int \frac{1}{x+7} \right) dx \\ &= \frac{1}{15} [\log|x-8| - \log|x+7|] + c \\ &= \frac{1}{15} \log \left| \frac{x-8}{x+7} \right| + c \end{aligned}$$

Choose the correct alternative from the following.

$$\int \left( x + \frac{1}{x} \right)^3 dx =$$

Options

$$\frac{1}{4} \left( x + \frac{1}{x} \right)^4 + c$$

$$\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x - \frac{1}{2x^2} + c$$

$$\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x + \frac{1}{x^2} + c$$

$$(x - x^{-1})^3 + c$$

**Solution:**

$$\frac{x^4}{4} + \frac{3x^2}{2} + 3 \log x - \frac{1}{2x^2} + c$$

**Explanation:**

$$\text{Let } I = \int \left( x + \frac{1}{x} \right)^3 dx$$

$$\int \left( x^3 + 3x + \frac{3}{x} + \frac{1}{x^3} \right) dx$$

$$= \frac{x^4}{4} + 3 \frac{x^2}{2} + 3 \log|x| - \frac{1}{2x^2} + c$$

Choose the correct alternative from the following.

$$\int \left( \frac{e^{2x} + e^{-2x}}{e^x} \right) dx =$$

Options

$$e^x - \frac{1}{3e^{3x}} + c$$

$$e^x + \frac{1}{3e^{3x}} + c$$

$$e^{-x} + \frac{1}{3e^{3x}} + c$$

$$e^{-x} + \frac{1}{3e^{3x}} + c$$

Solution:

$$e^x - \frac{1}{3e^{3x}} + c$$

Explanation:

$$\begin{aligned} \int \left( \frac{e^{2x} + e^{-2x}}{e^x} \right) dx &= \int (e^x + e^{-3x}) dx \\ &= e^x - \frac{1}{3} e^{-3x} + c \end{aligned}$$

Miscellaneous Exercise 5 | Q 1.09 | Page 137

Choose the correct alternative from the following.

$$\int (1-x)^{-2} dx =$$

1.  $(1+x)^{-1} + c$

2.  $(1-x)^{-1} + c$

3.  $(1-x)^{-1-1} + c$

4.  $(1-x)^{-1+1} + c$

**Solution:**

$$(1-x)^{-1} + c$$

**Explanation:**

$$\begin{aligned}\int (1-x)^{-2} dx &= \frac{(1-x)^{-1}}{-1 \times -1} + c \\ &= (1-x)^{-1} + c\end{aligned}$$

**Miscellaneous Exercise 5 | Q 1.1 | Page 138**

**Choose the correct alternative from the following.**

$$\int \frac{(x^3 + 3x^2 + 3x + 1)}{(x+1)^5} dx =$$

Options

$$\frac{-1}{x+1} + c$$

$$\left(\frac{-1}{x+1}\right)^5 + c$$

$$\log(x+1) + c$$

$$\log |x+1|^5 + c$$

**Solution:**

$$\frac{-1}{x+1} + c$$

**Explanation:**

$$\int \frac{(x^3 + 3x^2 + 3x + 1)}{(x+1)^5} dx = \int \frac{(x+1)^3}{(x+1)^5} dx$$



$$= \int \frac{1}{(x+1)} dx$$

$$= \frac{-1}{x+1} + c$$

**Miscellaneous Exercise 5 | Q 2.1 | Page 138**

**Fill in the Blank.**

$$\int \frac{5(x^6 + 1)}{x^2 + 1} dx = x^5 + \underline{\hspace{2cm}} x^3 + 5x + c$$

**Solution:**

$$\int \frac{5(x^6 + 1)}{x^2 + 1} dx = x^5 + \frac{-5}{3} x^3 + 5x + c$$

**Explanation:**

$$\int \frac{5(x^6 + 1)}{x^2 + 1} dx = 5 \int \frac{(x^2 + 1)(x^4 - x^2 + 1)}{x^2 + 1} dx$$

$$= 5 \int (x^4 - x^2 + 1) dx$$

$$= 5 \left( \frac{x^5}{5} - \frac{x^3}{3} + x \right) + c$$

$$= x^5 - \frac{5}{3} x^3 + 5x + c$$

**Miscellaneous Exercise 5 | Q 2.2 | Page 138**

**Fill in the Blank.**

$$\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx = x + \underline{\hspace{2cm}} + c$$

**Solution:**

$$\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx = x + \underline{4 \log|x-1|} + c$$

**Explanation:**

$$\begin{aligned}\int \frac{x^2 + x - 6}{(x-2)(x-1)} dx &= \int \frac{(x+3)(x-2)}{(x-2)(x-1)} dx \\&= \int \frac{x+3}{x-1} dx \\&= \int \frac{(x-1) + 4}{x-1} dx \\&= \int \left(1 + \frac{4}{x-1}\right) dx \\&= x + 4 \log |x-1| + c\end{aligned}$$

**Miscellaneous Exercise 5 | Q 2.3 | Page 138**

**Fill in the Blank.**

If  $f'(x) = \frac{1}{x} + x$  and  $f(1) = \frac{5}{2}$ , then  $f(x) = \log x + \frac{x^2}{2} + \underline{\hspace{1cm}}$

**Solution:**

If  $f'(x) = \frac{1}{x} + x$  and  $f(1) = \frac{5}{2}$ , then  $f(x) = \log x + \frac{x^2}{2} + \underline{2}$

**Explanation:**

$$\begin{aligned}f(x) &= \int f'(x) dx \\&= \int \left(\frac{1}{x} + x\right) dx \\f(x) &= \log |x| + \frac{x^2}{2} + c \quad \dots(i) \\f(1) &= \frac{5}{2}\end{aligned}$$

$$f(1) = \log 1 + \frac{1^2}{2} + c$$

$$\therefore \frac{5}{2} = 0 + \frac{1}{2} + c$$

$$\therefore c = 2$$

$$\therefore f(x) = \log |x| + \frac{x^2}{2} + 2$$

#### Miscellaneous Exercise 5 | Q 2.4 | Page 138

##### Fill in the Blank.

To find the value of  $\int \frac{(1 + \log x)dx}{x}$  the proper substitution is \_\_\_\_\_

**Solution:**

To find the value of  $\int \frac{(1 + \log x)dx}{x}$  the proper substitution is  **$1 + \log x = t$** .

#### Miscellaneous Exercise 5 | Q 2.5 | Page 138

##### Fill in the Blank.

$$\int \frac{1}{x^3} [\log x^x]^2 dx = P(\log x)^3 + c, \text{ then } P = \underline{\hspace{2cm}}$$

**Solution:**

$$\int \frac{1}{x^3} [\log x^x]^2 dx = P(\log x)^3 + c, \text{ then } P = \underline{\frac{1}{3}}$$

**Explanation:**

$$\text{Let } I = \int \frac{1}{x^3} [\log x^x]^2 dx = P \cdot (\log x)^3 + c$$

$$I = \int \frac{1}{x^3} [\log x^x]^2 dx = \int \frac{1}{x^3} (x \log x)^2 \cdot dx$$

$$= \int \frac{1}{x^3} \cdot x^2 \cdot (\log x)^2 dx = \int \frac{1}{x} (\log x)^2 \cdot dx$$

$\therefore$  Put  $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int t^2 \cdot dt$$

$$= \frac{t^3}{3} + c$$

$$= \frac{1}{3} (\log x)^3 + c$$

$$\therefore P = \frac{1}{3}$$

#### Miscellaneous Exercise 5 | Q 3.1 | Page 138

State whether the following statement is True or False.

The proper substitution for  $\int x (x^x)^x (2 \log x + 1) dx$  is  $(x^x)^x = t$

1. True

2. False

**Solution:** True

**Explanation:**

Let  $I = \int x (x^x)^x (2 \log x + 1) dx$

Put  $(x^x)^x = t$

Taking logarithm of both sides, we get

$$\log (x^x)^x = \log t$$

$$\therefore x^2 \cdot \log x = \log t$$

Differentiating w.r.t.  $x$ , we get

$$x^2 \cdot \frac{1}{x} + (\log x) \cdot 2x = \frac{1}{t} \cdot \frac{dt}{dx}$$

$$\therefore (x + 2x \log x) dx = \frac{1}{t} \cdot dt$$

$$\therefore x(1 + 2 \log x) dx = \frac{1}{t} \cdot dt$$

$$\therefore I = \int t \cdot \frac{1}{t} \cdot dt = \int 1 \cdot dt = t + c = (x^x)^x + c$$

### Miscellaneous Exercise 5 | Q 3.2 | Page 138

**State whether the following statement is True or False.**

If  $\int x e^{2x} dx$  is equal to  $e^{2x} f(x) + c$ , where  $c$  is constant of integration, then  $f(x)$  is  $\frac{2x-1}{2}$ .

1. True

2. False

**Solution:** False

**Explanation:**

$$\begin{aligned} \text{Let } I &= \int x \cdot e^{2x} dx \\ &= x \int e^{2x} \cdot dx - \int \left[ \frac{d}{dx}(x) \int e^{2x} \cdot dx \right] dx \\ &= x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} \cdot dx \\ &= \frac{x}{2} e^{2x} - \frac{1}{2} \int e^{2x} + c \\ &= \frac{x}{2} e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2} + c \end{aligned}$$

$$= e^{2x} \left( \frac{x}{2} - \frac{1}{4} \right) + c$$

$$= e^{2x} \left( \frac{2x - 1}{4} \right) + c$$

$$\therefore f(x) = \frac{2x - 1}{4}$$

### Miscellaneous Exercise 5 | Q 3.3 | Page 138

**State whether the following statement is True or False.**

If  $\int x f(x) dx = \frac{f(x)}{2}$ , then find  $f(x) = e^{x^2}$

1. True

2. False

**Solution:** True

**Explanation:**

If  $f(x) = e^{x^2}$ , then

$$\int x \cdot f(x) dx = \int x \cdot e^{x^2} \cdot dx$$

Put  $x^2 = t$

$$\therefore 2x dx = dt$$

$$\therefore x dx = \frac{1}{2} dt$$

$$\therefore \int x \cdot f(x) dx = \frac{1}{2} \int e^t \cdot dt$$

$$= \frac{1}{2} e^t + c$$

$$= \frac{1}{2} e^{x^2} + c$$

$$= \frac{1}{2} f(x) + c$$

**State whether the following statement is True or False.**

If  $\int \frac{(x-1)dx}{(x+1)(x-2)} = A \log |x+1| + B \log |x-2| + c$ , then  $A + B = 1$ .

1. True

2. False

**Solution:** True

**Explanation:**

$$\text{Let } I = \frac{(x-1)}{(x+1)(x-2)} dx$$

$$\text{Let } \frac{(x-1)}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$$

$$\therefore x-1 = A(x-2) + B(x+1) \quad \dots(i)$$

Putting  $x = -1$  in (i), we get

$$-1-1 = A(-1-2)$$

$$\therefore -2 = -3A$$

$$\therefore A = \frac{2}{3}$$

Putting  $x = 2$  in (i), we get

$$2-1 = B(2+1)$$

$$\therefore 1 = 3B$$

$$\begin{aligned} \therefore I &= \int \left( \frac{\frac{2}{3}}{x+1} + \frac{\frac{1}{3}}{x-2} \right) dx \\ &= \frac{2}{3} \int \frac{1}{x+1} dx + \frac{1}{3} \int \frac{1}{x-2} dx \end{aligned}$$

Comparing the above with

$A \log |x + 1| + B \log |x - 2| + c$ , we get

$$\therefore A = \frac{2}{3}, B = \frac{1}{3}$$

$$\therefore A + B = \frac{2}{3} + \frac{1}{3} = 1$$

#### Miscellaneous Exercise 5 | Q 3.5 | Page 138

**State whether the following statement is True or False.**

For  $\int \frac{x-1}{(x+1)^3} e^x dx = e^x f(x) + c$ ,  $f(x) = (x+1)^2$ .

1. True

2. False

**Solution:** False

**Explanation:**

$$\begin{aligned}\text{Let } I &= \frac{(x-1)}{(x+1)^3} \cdot e^x dx \\ &= \int e^x \left[ \frac{(x+1) - 2}{(x+1)^3} \right] dx \\ &= \int e^x \left[ \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3} \right] dx \\ &= \int e^x \left[ (x+1)^{-2} - 2(x+1)^{-3} \right] dx\end{aligned}$$

Put  $f(x) = (x+1)^{-2}$

$\therefore f'(x) = -2(x+1)^{-3}$



$$\therefore I = e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$= e^x \cdot (x + 1)^{-2} + c$$

$$\therefore f(x) = (x + 1)^{-2}$$

### Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate**  $\int \frac{5x^2 - 6x + 3}{2x - 3} dx$

**Solution:**

Let  $I = \int \frac{5x^2 - 6x + 3}{2x - 3} dx$

We perform actual division and express the result as:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\frac{5}{2}x + \frac{3}{4}$$

$$2x - 2 \overline{) 5x^2 - 6x + 3}$$

$$5x^2 - \frac{15}{2}x$$

$$\begin{array}{r} \text{(-)} \quad \text{(+)} \\ \hline \end{array}$$

$$\frac{3x}{2} + 3$$

$$\frac{3x}{2} - \frac{9}{4}$$

$$\frac{3x}{2} - \frac{9}{4}$$

$$\frac{3x}{2} - \frac{9}{4}$$

$$\begin{array}{r} \text{(-)} \quad \text{(+)} \\ \hline \end{array}$$

$$\frac{21}{4}$$

$$\frac{21}{4}$$

$$\begin{aligned}
 \therefore I &= \int \left( \frac{5}{2}x + \frac{3}{4} + \frac{\frac{21}{4}}{2x-2} \right) dx \\
 &= \frac{5}{2} \int x \, dx + \frac{3}{4} \int dx + \frac{21}{4} \int \frac{1}{2x-3} \, dx \\
 &= \frac{5}{2} \cdot \frac{x^2}{2} + \frac{3}{4}x + \frac{21}{4} \cdot \frac{\log|2x-3|}{2} + c \\
 \therefore I &= \frac{5x^2}{4} + \frac{3}{4}x + \frac{21 \log|2x-3|}{8} + c
 \end{aligned}$$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate**  $\int (5x+1)^{\frac{4}{9}} \, dx$

**Solution:**

$$\begin{aligned}
 \text{Let } I &= \int (5x+1)^{\frac{4}{9}} \, dx \\
 &= \frac{(5x+1)^{\frac{4}{9}+1}}{\left(\frac{4}{9}+1\right) \times 5} + c \\
 &= \frac{(5x+1)^{\frac{13}{9}}}{\frac{13}{9} \times 5} + c \\
 \therefore I &= \frac{9}{65} (5x+1)^{\frac{13}{9}} + c
 \end{aligned}$$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate**  $\int \frac{1}{(2x+3)} \, dx$

**Solution:**

$$\text{Let } I = \int \frac{1}{2x+3} dx$$

$$\therefore I = \frac{\log|2x+3|}{2} + c$$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate**  $\int \frac{x-1}{\sqrt{x+4}} dx$

**Solution:**

$$\text{Let } I = \int \frac{x-1}{\sqrt{x+4}} dx$$

$$= \int \frac{(x+4)-5}{\sqrt{x+4}} dx$$

$$= \int \left( \sqrt{x+4} - \frac{5}{\sqrt{x+4}} \right) dx$$

$$= \int \left[ (x+4)^{\frac{1}{2}} - 5(x+4)^{-\frac{1}{2}} \right] dx$$

$$= \frac{(x+4)^{\frac{3}{2}}}{\frac{3}{2}} - 5 \frac{(x+4)^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$\therefore I = \frac{2}{3}(x+4)^{\frac{3}{2}} - 10\sqrt{x+4} + c$$

Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate:** If  $f'(x) = \sqrt{x}$  and  $f(1) = 2$ , then find the value of  $f(x)$ .

**Solution:**

$$f'(x) = \sqrt{x} \quad \dots[\text{Given}]$$

$$f(x) = \int f'(x)$$

$$= \int \sqrt{x} \, dx$$

$$= \int x^{\frac{1}{2}} \, dx$$

$$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$$

$$\therefore f(x) = \frac{2}{3}x^{\frac{3}{2}} + c \dots(i)$$

$$\text{Now, } f(1) = 2 \quad \dots[\text{Given}]$$

$$\therefore \frac{2}{3}(1)^{\frac{3}{2}} + c = 2$$

$$\therefore c = 2 - \frac{2}{3}$$

$$\therefore c = \frac{4}{3}$$

$$\therefore f(x) = \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$$

### Miscellaneous Exercise 5 | Q 4.1 | Page 138

**Evaluate:**  $\int |x| \, dx$  if  $x < 0$

**Solution:**  $|x| = x; x \geq 0$

$= x; x < 0$

Let  $I = \int |x| \, dx$ , if  $x < 0$

$= \int -x \, dx$

$$\therefore I = \frac{-x^2}{2} + c$$

Miscellaneous Exercise 5 | Q 4.2 | Page 138

**Evaluate:** Find the primitive of  $\frac{1}{1 + e^x}$

**Solution:**

$$\text{Let } I = \int \frac{1}{1 + e^x} dx$$

Dividing Nr. and Dr. by  $e^x$ , we get

$$I = \int \frac{e^{-x}}{e^{-x} + 1} dx$$

$$\text{Put } e^{-x} + 1 = t$$

$$\therefore -e^{-x}dx = dt$$

$$\therefore e^{-x}dx = -dt$$

$$\therefore I = \int \frac{-dt}{t} = -\log|t| + c$$

$$\therefore I = -\log|e^{-x} + 1| + c$$

Miscellaneous Exercise 5 | Q 4.2 | Page 138

**Evaluate:**  $\int \frac{ae^x + be^{-x}}{(ae^x - be^{-x})} dx$

**Solution:**

$$\text{Let } I = \int \frac{ae^x + be^{-x}}{(ae^x - be^{-x})} dx$$

$$\text{Put } ae^x - be^{-x} = t$$

$$\therefore (ae^x + be^{-x})dx = dt$$

$$\therefore I = \int \frac{dt}{t} = \log|t| + c$$

$$\therefore I = \log |ae^x + be^{-x}| + c$$

**Miscellaneous Exercise 5 | Q 4.2 | Page 138**

**Evaluate:**  $\int \frac{1}{2x + 3x \log x} dx$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int \frac{1}{2x + 3x \cdot \log x} dx \\ &= \int \frac{1}{x(2 + 3 \log x)} dx \end{aligned}$$

$$\text{Put } 2 + 3 \log x = t$$

$$\therefore 3 \cdot \frac{1}{x} dx = dt$$

$$\therefore \frac{1}{x} dx = \frac{1}{3} dt$$

$$\therefore I = \frac{1}{3} \int \frac{1}{t} \cdot dt$$

$$= \frac{1}{3} \log |t| + c$$

$$\therefore I = \frac{1}{3} \log |2 + 3 \log x| + c$$

**Miscellaneous Exercise 5 | Q 4.2 | Page 138**

**Evaluate:**  $\int \frac{1}{\sqrt{x} + x} dx$

**Solution:**

$$\text{Let } I = \int \frac{1}{\sqrt{x} + x} dx$$

$$= \int \frac{1}{\sqrt{x}(1 + \sqrt{x})} dx$$

Put  $1 + \sqrt{x} = t$

$$\therefore \frac{1}{2\sqrt{x}} dx = dt$$

$$\therefore \frac{1}{\sqrt{x}} dx = 2 dt$$

$$\therefore I = \int \frac{2 \cdot dt}{t}$$

$$= 2 \int \frac{1}{t} dt$$

$$= 2 \log |t| + c$$

$$\therefore I = 2 \log |1 + \sqrt{x}| + c$$

#### Miscellaneous Exercise 5 | Q 4.2 | Page 138

**Evaluate:**  $\int \frac{2e^x - 3}{4e^x + 1} dx$

**Solution:**

Let  $I = \int \frac{2e^x - 3}{4e^x + 1} dx$

Let  $2e^x - 3 = A(4e^x + 1) + B \frac{d}{dx}(4e^x + 1)$

$$\therefore 2e^x - 3 = (4A + 4B)e^x + A$$

Comparing the coefficients of  $e^x$  and constant term on both sides, we get

$$4A + 4B = 2 \text{ and } A = -3$$

Solving these equations, we get

$$B = \frac{7}{2}$$

$$\therefore I = \frac{-3(4e^x + 1) + \frac{7}{2}(4e^x)}{4e^x + 1} dx$$

$$= -3 \int dx + \frac{7}{2} \int \frac{4e^x}{4e^x + 1} dx$$

$$\therefore I = -3x + \frac{7}{2} \log|4e^x + 1| + c \quad \dots \left[ \because \int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c \right]$$

#### Miscellaneous Exercise 5 | Q 4.3 | Page 138

**Evaluate:**  $\int \frac{dx}{\sqrt{4x^2 - 5}}$

**Solution:**

$$\text{Let } I = \int \frac{dx}{\sqrt{4x^2 - 5}}$$

$$= \int \frac{1}{\sqrt{4\left(x^2 - \frac{5}{4}\right)}} dx$$

$$= \frac{1}{2} \int \frac{1}{\sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2}} dx$$

$$= \frac{1}{2} \log \left| x + \sqrt{x^2 - \left(\frac{\sqrt{5}}{2}\right)^2} \right| + c$$

$$\therefore I = \frac{1}{2} \log \left| x + \sqrt{x^2 - \frac{5}{4}} \right| + c$$

#### Miscellaneous Exercise 5 | Q 4.3 | Page 138



**Evaluate:**  $\int \frac{dx}{3 - 2x - x^2}$

**Solution:**

Let  $I = \int \frac{dx}{3 - 2x - x^2}$

$$3 - 2x - x^2 = -x^2 - 2x + 3$$

$$= -(x^2 + 2x - 3)$$

$$= -(x^2 + 2x + 1 - 4)$$

$$= -[(x + 1)^2 - 4]$$

$$= (2)^2 - (x + 1)^2$$

$$\therefore I = \int \frac{dx}{(2)^2 - (x + 1)^2}$$

$$= \frac{1}{2(2)} \log \left| \frac{2 + x + 1}{2 - (x + 1)} \right| + c$$

$$\therefore I = \frac{1}{4} \log \left| \frac{3 + x}{1 - x} \right| + c$$

**Miscellaneous Exercise 5 | Q 4.3 | Page 138**

**Evaluate:**  $\int \frac{dx}{9x^2 - 25}$

**Solution:**

Let  $I = \int \frac{dx}{9x^2 - 25}$

$$= \int \frac{1}{9\left(x^2 - \frac{25}{9}\right)} dx$$

$$\begin{aligned}
 &= \frac{1}{9} \int \frac{1}{x^2 - \left(\frac{5}{3}\right)^2} dx \\
 &= \frac{1}{9} \cdot \frac{1}{2 \cdot \frac{5}{3}} \log \left| \frac{x - \frac{5}{3}}{x + \frac{5}{3}} \right| + c \\
 \therefore I &= \frac{1}{30} \log \left| \frac{3x - 5}{3x + 5} \right| + c
 \end{aligned}$$

#### Miscellaneous Exercise 5 | Q 4.3 | Page 139

**Evaluate:**  $\int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}} dx$

**Solution:**

$$\begin{aligned}
 \text{Let } I &= \int \frac{e^x}{\sqrt{e^{2x} + 4e^x + 13}} dx \\
 &= \int \frac{e^x}{\sqrt{(e^x)^2 + 4e^x + 13}} dx
 \end{aligned}$$

Put  $e^x = t$

$\therefore e^x dx = dt$

$$\begin{aligned}
 \therefore I &= \int \frac{dt}{\sqrt{t^2 + 4t + 13}} \\
 &= \int \frac{1}{\sqrt{t^2 + 4t + 4 - 4 + 13}} dt \\
 &= \int \frac{1}{\sqrt{(t+2)^2 + 9}} dt \\
 &= \int \frac{1}{\sqrt{(t+2)^2 + (3)^2}} dt
 \end{aligned}$$

$$= \log \left| t + 2 + \sqrt{(t + 2)^2 + (3)^2} \right| + c$$

$$= \log \left| t + 2 + \sqrt{t^2 + 4t + 13} \right|$$

$$\therefore I = \log \left| e^x + 2 + \sqrt{e^{2x} + 4e^x + 13} \right|$$

#### Miscellaneous Exercise 5 | Q 4.3 | Page 139

**Evaluate:**  $\int \frac{dx}{x[(\log x)^2 + 4 \log x - 1]}$

**Solution:**

Let  $I = \int \frac{dx}{x[(\log x)^2 + 4 \log x - 1]}$

Put  $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int \frac{dt}{t^2 + 4t - 1}$$

$$= \int \frac{1}{t^2 + 4t + 4 - 4 - 1} dt$$

$$= \int \frac{1}{(t + 2)^2 - 5} dt$$

$$= \int \frac{1}{(t + 2)^2 - (\sqrt{5})^2} dt$$

$$= \frac{1}{2\sqrt{5}} \log \left| \frac{t + 2 - \sqrt{5}}{t + 2 + \sqrt{5}} \right| + c$$

$$\therefore I = \frac{1}{2\sqrt{5}} \log \left| \frac{\log x + 2 - \sqrt{5}}{\log x + 2 + \sqrt{5}} \right| + c$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

**Evaluate:**  $\int \frac{dx}{5 - 16x^2}$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{5 - 16x^2} \\ &= \int \frac{1}{16\left(\frac{5}{16} - x^2\right)} dx \\ &= \frac{1}{16} \int \frac{1}{\left(\frac{\sqrt{5}}{4}\right)^2 - x^2} dx \\ &= \frac{1}{16} \cdot \frac{1}{2\frac{\sqrt{5}}{4}} \log \left| \frac{\frac{\sqrt{5}}{4} + x}{\frac{\sqrt{5}}{4} - x} \right| + c \\ \therefore I &= \frac{1}{8\sqrt{5}} \log \left| \frac{\sqrt{5} + 4x}{\sqrt{5} - 4x} \right| + c \end{aligned}$$

Miscellaneous Exercise 5 | Q 4.3 | Page 139

**Evaluate:**  $\int \frac{dx}{25x - x(\log x)^2}$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int \frac{dx}{25x - x(\log x)^2} \\ &= \int \frac{1}{x[25 - (\log x)^2]} dx \end{aligned}$$

Put  $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int \frac{dt}{25 - t^2}$$

$$= \int \frac{1}{(5)^2 - t^2} dt$$

$$= \frac{1}{2(5)} \cdot \log \left| \frac{5+t}{5-t} \right| + c$$

$$\therefore I = \frac{1}{10} \log \left| \frac{5 + \log x}{5 - \log x} \right| + c$$

#### Miscellaneous Exercise 5 | Q 4.3 | Page 139

**Evaluate:**  $\int \frac{e^x}{4e^{2x} - 1} dx$

**Solution:**

$$\text{Let } I = \int \frac{e^x}{4e^{2x} - 1} dx$$

$$= \int \frac{e^x}{4(e^x)^2 - 1} dx$$

Put  $e^x = t$

$$\therefore e^x dx = dt$$

$$\therefore I = \int \frac{dt}{4t^2 - 1}$$

$$= \frac{1}{4} \int \frac{1}{t^2 - \frac{1}{4}} dx$$

$$= \frac{1}{4} \int \frac{1}{t^2 - \left(\frac{1}{2}\right)^2} dt$$

$$= \frac{1}{4} \cdot \frac{1}{2\left(\frac{1}{2}\right)} \log \left| \frac{t - \frac{1}{2}}{t + \frac{1}{2}} \right| + c$$

$$= \frac{1}{4} \log \left| \frac{2t - 1}{2t + 1} \right| + c$$

#### Miscellaneous Exercise 5 | Q 4.4 | Page 139

**Evaluate:**  $\int (\log x)^2 dx$

**Solution:**

$$\begin{aligned} \text{Let } I &= \int (\log x)^2 dx \\ &= \int (\log x)^2 \cdot 1 dx \\ &= (\log x)^2 \int 1 \cdot dx - \int \left[ \frac{d}{dx} (\log x)^2 \int 1 \cdot dx \right] dx \\ &= x(\log x)^2 - \int 2 \log x \cdot \frac{1}{x} \cdot x \cdot dx \\ &= x(\log x)^2 - 2 \int (\log x) \cdot 1 \cdot dx \\ &= x(\log x)^2 - 2 \left[ \log x \int 1 \cdot dx - \int \left\{ \frac{d}{dx} (\log x) \int 1 \cdot dx \right\} dx \right] \\ &= x(\log x)^2 - 2 \left[ (\log x)x - \int \frac{1}{x} \cdot x \cdot dx \right] \\ &= x(\log x)^2 - 2 \left[ x \log x - \int 1 \cdot dx \right] \\ &= x(\log x)^2 - 2(x \log x - x) + c \end{aligned}$$

$$\therefore I = x(\log x)^2 - 2x \log x - 2x + c$$

Miscellaneous Exercise 5 | Q 4.4 | Page 139

**Evaluate:**  $\int e^x \frac{1+x}{(2+x)^2} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int e^x \frac{1+x}{(2+x)^2} dx \\ &= \int e^x \left[ \frac{(2+x) - 1}{(2+x)^2} \right] dx \\ &= \int e^x \left[ \frac{1}{2+x} - \frac{1}{(2+x)^2} \right] dx\end{aligned}$$

$$\text{Let } f(x) = \frac{1}{2+x}$$

$$\therefore f'(x) = \frac{-1}{(2+x)^2}$$

$$\therefore I = \int e^x [f(x) + f'(x)] dx$$

$$= e^x \cdot f(x) + c$$

$$= e^x \cdot \frac{1}{2+x} + c$$

$$\therefore I = \frac{e^x}{2+x} + c$$

Miscellaneous Exercise 5 | Q 4.4 | Page 139

**Evaluate:**  $\int x \cdot e^{2x} dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int x \cdot e^{2x} dx \\&= x \int e^{2x} dx - \int \left[ \frac{d}{dx}(x) \int e^{2x} \cdot dx \right] dx \\&= x \cdot \frac{e^{2x}}{2} - \int 1 \cdot \frac{e^{2x}}{2} dx \\&= \frac{1}{2} x e^{2x} - \frac{1}{2} \int e^{2x} dx \\&= \frac{1}{2} x e^{2x} - \frac{1}{2} \cdot \frac{e^{2x}}{2} + c \\ \therefore I &= \frac{1}{4} e^{2x} (2x - 1) + c\end{aligned}$$

**Miscellaneous Exercise 5 | Q 4.4 | Page 139**

**Evaluate:**  $\int \log(x^2 + x) dx$

**Solution:**

$$\begin{aligned}\text{Let } I &= \int \log(x^2 + x) dx \\&= \int \log(x^2 + x) \cdot 1 \cdot dx \\&= \log(x^2 + x) \int 1 \cdot dx - \int \left\{ \frac{d}{dx} \log(x^2 + x) \int 1 \cdot dx \right\} dx \\&= \log(x^2 + x) \cdot x - \int \frac{1}{x^2 + x} \cdot (2x + 1) \cdot x \cdot dx \\&= x \cdot \log(x^2 + x) - \int \frac{1}{x(x + 1)} \cdot (2x + 1) \cdot x \cdot dx \\&= x \cdot \log(x^2 + x) - \int \frac{2x + 1}{x + 1} dx\end{aligned}$$



$$\begin{aligned}
&= x \cdot \log(x^2 + x) - \int \frac{(2x + 2) - 1}{x + 1} dx \\
&= x \cdot \log(x^2 + x) - \int \left[ \frac{2(x + 1)}{x + 1} - \frac{1}{x + 1} \right] dx \\
&= x \cdot \log(x^2 + x) - \int \left[ 2 - \frac{1}{x + 1} \right] dx \\
&= x \cdot [\log(x^2 + x)] - (2x - \log|x + 1|) + c \\
\therefore I &= x \cdot [\log(x^2 + x)] - 2x + \log|x + 1| + c
\end{aligned}$$

**Miscellaneous Exercise 5 | Q 4.4 | Page 139**

**Evaluate:**  $\int e^{\sqrt{x}} dx$

**Solution:**

Let  $I = \int e^{\sqrt{x}} dx$

Put  $\sqrt{x} = t$

$\therefore x = t^2$

$\therefore dx = 2t dt$

$\therefore I = \int e^t \cdot 2t dt$

$= 2 \int t \cdot e^t \cdot dt$

$= 2 \left[ t \int e^t dt - \int \left\{ \frac{d}{dx}(t) \int e^t \cdot dt \right\} dt \right]$

$= 2 \left[ t \cdot e^t - \int 1 \cdot e^t dt \right]$

$$= 2(te^t - e^t) + c$$

$$= 2e^t(t - 1) + c$$

$$\therefore I = 2e^{\sqrt{x}}(\sqrt{x} - 1) + c$$

**Miscellaneous Exercise 5 | Q 4.4 | Page 139**

**Evaluate:**  $\int \sqrt{x^2 + 2x + 5} \, dx$

**Solution:**

$$\text{Let } I = \int \sqrt{x^2 + 2x + 5} \, dx$$

$$= \int \sqrt{x^2 + 2x + 1 + 4} \, dx$$

$$= \int \sqrt{(x + 1)^2 + (2)^2} \, dx$$

$$= \frac{x + 1}{2} \sqrt{(x + 1)^2 + (2)^2} + \frac{(2)^2}{2} \log \left| (x + 1) + \sqrt{(x + 1)^2 + (2)^2} \right| + c$$

$$\therefore I = \frac{x + 1}{2} \sqrt{x^2 + 2x + 5} + 2 \log \left| (x + 1) + \sqrt{x^2 + 2x + 5} \right| + c$$

**Miscellaneous Exercise 5 | Q 4.4 | Page 139**

**Evaluate:**  $\int \sqrt{x^2 - 8x + 7} \, dx$

**Solution:**

$$\text{Let } I = \int \sqrt{x^2 - 8x + 7} \, dx$$

$$= \int \sqrt{x^2 - 8x + 16 - 9} \, dx$$

$$= \int \sqrt{(x - 4)^2 + (3)^2} \, dx$$

$$= \frac{x-4}{2} \sqrt{(x-4)^2 - (3)^2} - \frac{(3)^2}{2} \log \left| (x-4) + \sqrt{(x-4)^2 - (3)^2} \right| + c$$

$$\therefore I = \frac{x-4}{2} \sqrt{x^2 - 8x + 7} + \frac{9}{2} \log \left| (x-4) + \sqrt{x^2 - 8x + 7} \right| + c$$

### Miscellaneous Exercise 5 | Q 4.5 | Page 139

**Evaluate:**  $\int \frac{3x-1}{2x^2-x-1} dx$

**Solution:**

$$\text{Let } I = \int \frac{3x-1}{2x^2-x-1} dx$$

$$= \int \frac{3x-1}{(x-1)(2x+1)} dx$$

$$\text{Let } \frac{3x-1}{(x-1)(2x+1)} = \frac{A}{x-1} + \frac{B}{2x+1}$$

$$\therefore 3x-1 = A(2x+1) + B(x-1) \quad \dots(i)$$

Putting  $x = 1$  in (i), we get

$$3(1) - 1 = A(2+1) + B(0)$$

$$\therefore 2 = 3A$$

$$\therefore A = \frac{2}{3}$$

Putting  $x = -\frac{1}{2}$  in (i), we get

$$3\left(-\frac{1}{2}\right) - 1 = A(0) + B\left[-\frac{1}{2} - 1\right]$$

$$\therefore -\frac{5}{2} = B\left(-\frac{3}{2}\right)$$

$$\therefore B = \frac{5}{3}$$

$$\therefore \frac{3x - 1}{(x - 1)(2x + 1)} = \frac{\frac{2}{3}}{x - 1} + \frac{\frac{5}{3}}{2x + 1}$$

$$\therefore I = \int \left( \frac{\frac{2}{3}}{x - 1} + \frac{\frac{5}{3}}{2x + 1} \right) dx$$

$$= \frac{2}{3} \int \frac{1}{x - 1} dx + \frac{5}{3} \int \frac{1}{2x + 1} dx$$

$$\therefore I = \frac{2}{3} \log|x - 1| + \frac{5}{3} \frac{\log|(2x + 1)|}{2} + c$$

#### Miscellaneous Exercise 5 | Q 4.5 | Page 139

**Evaluate:**  $\int \frac{2x^3 - 3x^2 - 9x + 1}{2x^2 - x - 10} dx$

**Solution:**

We perform actual division and express the result as:

$$\frac{\text{Dividend}}{\text{Divisor}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Divisor}}$$

$$\begin{array}{r} \phantom{2x^2 - x - 10} \overline{) 2x^3 - 3x^2 - 9x + 1} \\ \underline{2x^3 - x^2 - 10x} \phantom{+ 1} \\ (-) \phantom{2x^3} (+) \phantom{- 10x} \phantom{+ 1} \phantom{+ 1} \\ \underline{-2x^2 + x + 1} \phantom{+ 1} \\ -2x^2 + x + 10 \\ \underline{-(+)(-)(-)} \phantom{+ 1} \\ \phantom{-2x^2 + x} - 9 \end{array}$$

$$\therefore I = \int \left( x - 1 + \frac{-9}{2x^2 - x - 10} \right) dx$$

$$= \int x \cdot dx - \int 1 \cdot dx - 9 \int \frac{1}{2x^2 - x - 10} dx$$

Here  $2x^2 - x - 10$

$$= 2 \left( x^2 + \frac{1}{2}x + \frac{1}{16} - 5 - \frac{1}{16} \right)$$

$$= 2 \left[ \left( x - \frac{1}{4} \right)^2 - \frac{81}{16} \right]$$

$$\therefore I = \int x \cdot dx - \int 1 \cdot dx - \frac{9}{2} \int \frac{1}{\left( x - \frac{1}{4} \right)^2 - \left( \frac{9}{4} \right)^2} dx$$

$$= \frac{x^2}{2} - x - \frac{9}{2} \cdot \frac{1}{2\left(\frac{9}{4}\right)} \log \left| \frac{x - \frac{1}{4} - \frac{9}{4}}{x - \frac{1}{4} + \frac{9}{4}} \right| + c_1$$

$$= \frac{x^2}{2} - x - \log \left| \frac{x - \frac{5}{2}}{x + 2} \right| + c_1$$

$$= \frac{x^2}{2} - x - \log \left| \frac{2x - 5}{2(x + 2)} \right| + c_1$$

$$= \frac{x^2}{2} - x + \log \left| \frac{2(x + 2)}{2x - 5} \right| + c_1$$

$$= \frac{x^2}{2} - x + \log \left| \frac{x + 2}{2x - 5} \right| + \log 2 + c_1$$

$$\therefore I = \frac{x^2}{2} - x + \log \left| \frac{x + 2}{2x - 5} \right| + c \text{ where } c = c_1 + \log 2$$

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**Evaluate:**  $\int \frac{1 + \log x}{x(3 + \log x)(2 + 3 \log x)} dx$

**Solution:**

$$\text{Let } I = \int \frac{1 + \log x}{x(3 + \log x)(2 + 3 \log x)} dx$$

Put  $\log x = t$

$$\therefore \frac{1}{x} dx = dt$$

$$\therefore I = \int \frac{1 + t}{(3 + t)(2 + 3t)} dt$$

$$\text{Let } \frac{1 + t}{(3 + t)(2 + 3t)} = \frac{A}{3 + t} + \frac{B}{2 + 3t}$$

$$\therefore 1 + t = A(2 + 3t) + B(3 + t) \quad \dots(i)$$

Putting  $t = -3$  in (i), we get

$$1 - 3 = A(2 - 9) + B(0)$$

$$\therefore -2 = A(-7)$$

$$\therefore A = \frac{2}{7}$$

Putting  $t = -\frac{2}{3}$  in (i), we get

$$1 - \frac{2}{3} = A(0) + B\left(3 - \frac{2}{3}\right)$$

$$\therefore \frac{1}{3} = B\left(\frac{7}{3}\right)$$

$$\therefore B = \frac{1}{7}$$

$$\therefore \frac{1+t}{(3+t)(2+3t)} = \frac{\frac{2}{7}}{3+t} + \frac{\frac{1}{7}}{2+3t}$$

$$\therefore I = \int \left( \frac{\frac{2}{7}}{3+t} + \frac{\frac{1}{7}}{2+3t} \right) dt$$

$$= \frac{2}{7} \int \frac{1}{3+t} dt + \frac{1}{7} \int \frac{1}{2+3t} dt$$

$$= \frac{2}{7} \log|3+t| + \frac{1}{7} \cdot \frac{\log|2+3t|}{3} + c$$

$$\therefore I = \frac{2}{7} \log|3 + \log x| + \frac{1}{21} \log|2 + 3 \log x| + c$$